Allocative efficiency measurement revisited—Do we really need input prices?☆

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Abstract

The traditional approach to measuring allocative efficiency exploits input prices, which are rarely known at the firm level. This paper proves allocative efficiency can be measured as a profit-oriented distance to the frontier in a profit-technical efficiency space. This new approach does not require information on input prices. To validate the new approach, we perform a Monte-Carlo experiment providing evidence that the estimates of allocative efficiency employing the new and the traditional approach are highly correlated. Finally, as an illustration, we apply the new approach to a sample of about 900 enterprises from the chemical manufacturing industry in Germany.

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1. Introduction

A significant number of empirical studies have investigated the extent and determinants of technical efficiency within and across industries (e.g., Alvarez and Crespi, 2003; Caves and Barton, 1990; Gumbau-Albert and Maudos, 2002; Green and Mayes, 1991; Fritsch and Stephan, 2004). Comprehensive literature reviews of the variety of empirical applications are made by Lovell (1993) and Seiford (1996, 1997). Compared to this literature, attempts to

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2 Seiford (1996, 1997) compiles over 800 published articles and dissertations written only up to 1995. Sarafoglou (1998) estimated that bibliography to include approximately 700 articles in refereed journals, the others being dissertations, proceedings and/or book chapters. Yet, a 2002 bibliography lists more than 3000 references, 1200 of these are published in good quality journals (see Tavares, 2002), Gattoufi et al. (2004) claims that the literature base consists of over 1800 articles in refereed journals worldwide in addition to many books, conference proceedings, and various types of monographs.
quantify the extent and distribution of allocative efficiency are relatively rare (e.g., see a survey by Greene (1997)). Many more studies analyze the financial sector (see the review by Berger and Humphrey (1997) and also Topuz et al. (2005), Färe et al. (2004), Isik and Hassan (2002)). Some studies have been performed for the agricultural sector (e.g., Coelli et al., 2002; Chavas and Aliber, 1993; Chavas et al., 2005; Grazhdaninova and Zvi, 2005). Studies for manufacturing sector are relatively rare (e.g., Burki et al., 1997; Kim and Gwangho, 2001). This is quite surprising since allocative efficiency has traditionally attracted the attention of economists and business managers: what is the optimal combination of inputs so that output is produced at minimal cost? How much could the profits be increased by simply reallocating resources? To what extent does competitive pressure reduce the heterogeneity of allocative inefficiency within industries? Moreover, allocative efficiency is also important for the analysis of the production process; e.g., to estimate the bias of (i) the cost function parameters, (ii) returns to scale, (iii) input price elasticities, and (iv) cost-inefficiency (Kumbhakar and Wang, 2006) or to validate the aggregation of productivity index (Raa, 2005).

A firm is said to have realized allocative efficiency if it is operating with the optimal combination of inputs given input prices. The traditional approaches to measuring allocative efficiency require input prices (see Atkinson and Cornwell, 1994; Greene, 1997; Kumbhakar, 1991; Kumbhakar and Tsionas, 2005; Oum and Zhang, 1995) which are hardly available in reality. This explains why empirical studies of allocative efficiency are highly concentrated on certain industries, particularly banking, because information on input price can be readily obtained for these industries.

Recently, more attempts to measure allocative efficiency have been advanced. For example given incomplete input price data information, Kuosmanen and Post (2001) put forward a methodology to derive the upper and lower bounds of overall efficiency. Their technique makes an additional assumption that prices are within a convex polyhedral cone. Kuosmanen et al. (2006) continue by taking advantage of the law of one price, which is theoretically asserted to hold in the competitive market. The authors, while acknowledging that the input price vector is often not observed, consider the whole price domain instead of separate single input price vectors. The derived domain includes relative price restrictions, which allow them to obtain an upper bound of efficiency via maximization in terms of input prices that are endogenously chosen from the domain. Camanho and Dyson (2005) and later Jahanshahloo et al. (2008) have developed ways to measure upper and lower bounds for allocative efficiency when some information of input prices is known.

The major virtue of current article is that it introduces a new approach to estimating allocative efficiency, which is solely based on input and output quantities and profits and does not require information on input prices. An indicator for allocative efficiency is derived as the profit-oriented distance to a frontier in a profit-technical efficiency space.

What is, however, needed is an assessment of input-saving technical efficiency; i.e., how less input could be used to produce given outputs. This new approach rests on an assumption of the law of one price, or competitive market equilibrium in which all firms face the same input and output prices. The possibility of explore allocative efficiency as a part of overall efficiency when prices are the same across all firms was studied by Walter Briec and Leleu (2003), Li and Ng (1995), and Ylvinger (2000). However, one way or another, these studies use inputs prices to retrieve allocative efficiency measure. What is remarkable about the approach we propose is that it does not use them explicitly.

The paper unfolds as follows: Section 2 theoretically derives a new method for estimating allocative efficiency and introduces a theoretical framework for activity analysis models. Section 3 presents the results of the Monte-Carlo experiment on comparison of allocative efficiency scores calculated using both traditional and new approaches. Section 4 provides a rationale and a simple illustration using the new approach; Section 5 concludes.

2. Measurement of allocative efficiency

2.1. Traditional approach to allocative efficiency measurement

In his seminal paper, Farrell (1957) used the concept of efficiency postulated by Koopmans (1951) and the radial type of efficiency measure considered by Debreu (1951) to introduce the foundation for efficiency analysis. He differentiated between technical and allocative efficiencies. A firm is technically efficient if it uses the minimal possible combination of inputs for producing a certain output (input orientation). Allocative efficiency, or as Farrell called it price efficiency, refers to the ability of a firm to choose the optimal combination of inputs given input prices. If a firm has realized both technical and allocative efficiency, it is then cost efficient (overall efficient).
Fig. 1, similar to Kumbhakar and Lovell (2003), shows firm A producing output $y^A$ represented by the isoquant $L(y^A)$. Dotted lines are the isocosts which show level of expenditures for a certain combination of inputs. The slope of the isocosts is equal to the negative of ratio of input prices, $w(w_1, w_2)$. If the firm is producing output $y^A$ with the factor combination $x^A$ (a in Fig. 1), it is operating technically inefficient. Potentially, it could have produced the same output contracting both inputs $x_1$ and $x_2$ (available at prices $w$), proportionally (radial approach); the smallest possible contraction is in point $b$, representing $(\theta x^A)$ factor combination. Having reached this point, the firm is considered to be technically efficient. Formally, technical efficiency is measured by the ratio of the lowest attainable input level for producing a given amount of output to the current input level. In terms of Fig. 1, technical efficiency of unit $x^A$ is given by

$$TE(y^A, x^A) = \theta = \frac{w(\theta x^A)}{wx^A},$$

(1)

or geometrically by $0b/0a$. The measure of cost efficiency (overall efficiency) is given by the ratio of potentially minimal costs to actual costs:

$$CE(y^A, x^A, w^A) = \frac{wx^E}{wx^A},$$

(2)

or geometrically by $0c/0a$. Thus, cost efficiency is the ratio of costs at $x^E$ to costs at $x^A$ while technical efficiency is the ratio of costs at $(\theta x^A)$ to costs at $x^A$. The remaining portion of the cost efficiency is given by the ratio of costs at $x^E$ to costs at $(\theta x^A)$. It is attributable to the misallocation of inputs given input prices and is known as allocative efficiency:

$$AE = \frac{CE}{TE} = \frac{wx^E/wx^A}{w(\theta x^A)/wx^A},$$

(3)

or is given geometrically in terms of Fig. 1 by $0c/0b$.

2.2. A new approach to allocative efficiency measurement

When input prices are available, allocative efficiency in the pure Farrell sense can be calculated using, for example, a non-parametric frontier approach (Färe et al., 1994) or a parametric one (see Greene, 1997 among others). However, if input prices are not available these approaches are not applicable. In contrast to this, the new approach we propose allows measuring allocative efficiency without information on input prices. An estimate of allocative efficiency can be obtained with the new approach that is solely based on information on input and output quantities and on profits. The
first step of this new approach involves the estimation of technical efficiency; whereby, in the second step allocative
efficiency is estimated as a profit-oriented distance to the frontier in a profit-technical efficiency space.

Several assumptions are needed to prove our conjecture.

A1. The technology is homogeneous of degree 1: \( f(tx) = tf(x) \) (see Varian, 1992, p. 18).

A2. Firms face equal input prices \( w \) and equal output prices \( p \).

A3. Profit is expressed in relative terms in order to prevent size effects.\(^4\)

**Proposition 1.** Existence of the frontier in profit-technical efficiency space. A profit maximum exists for any level of
technical efficiency.

**Proof.** In Fig. 2, three firms, A, B, and C using inputs \( x^A \), \( x^B \) and \( x^C \), available at prices \( w \), produce output \( y^A \), which is
measured by the isoquant \( L(y^A) \). We will relax the assumption that firms are producing the same output shortly. For the
sake of argument, firms A, B, and C are all equally technically efficient (the level of technical efficiency \( \theta \), however, is
arbitrarily chosen), which is read from expenditure levels at \( (\theta x^A) \), \( (\theta x^B) \), and \( (\theta x^C) \), respectively. In geometrical terms
\( \theta x^A/0a^A = \theta x^B/0a^B = \theta x^C/0a^C \). The costs of these three firms are determined by \( wx^A \), \( wx^B \), and by \( wx^C \). The isocost
corresponding to expenditures at \( x^C \) is the closest possible to the origin 0 for this level of technical efficiency and, therefore,
implies the lowest level of costs. This is because \( x^C \) is the combination of inputs lying on the ray from origin and going
through the tangent point of the isocost (corresponding to cost level of \( wx^F \)) to the isoquant \( L(y^A) \). This implies that for \( \theta \)-
level of technical efficiency costs have a lower bound and using the fact that firms are producing the same output \( y^A \), profit
has an upper bound, given Assumptions A2 and A3. Without loss of generality, for each level \( \theta \) of technical efficiency
there is a profit maximum, which proves the existence of a frontier in profit-technical efficiency space. \( \square \)

**Remark 1.** Proposition 1 is valid for different outputs.

**Proof.** In Fig. 3, two firms, A and C (from Fig. 2) produce output \( y^A \); two other firms, F and G, use inputs \( x^F \) and \( x^G \) to
produce output \( y^B \), which is measured by the isoquant \( L(y^B) \). Output levels \( y^A \) and \( y^B \) are chosen arbitrarily. All four
firms are equally technically efficient. Let us compare profits of firms A and F, and of firms C and G.

At the output level \( y^A \) firm C has the largest profit, whereas firm G has the largest profit at the output level \( y^B \). Given
Assumptions A1, A2, and A3 the profit of firm A is equal to the profit of firm F. This is due to the fact that revenues and
costs increase by the same proportion as inputs do (Assumption A1). By the same logic, the profits of firms C and G are
equal. Thus, despite producing different output both firms C and G are on frontier defined in Proposition 1. \( \square \)

Note that this remark holds for any set of different output levels.

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\(^4\) In our theoretical derivation, in the Monte-Carlo study, and in the empirical illustration we define profit as follows: profit=(revenues–costs)/
revenues.
Remark 2. Frontier in profit-technical efficiency space is sloped upwards.

Proof. In Fig. 4, two firms, C and D, use inputs $x^C$ and $x^D$ to produce output $y^A$, which is measured by the isoquant $L(y^A)$. Both firms are allocatively efficient because they lie on the same ray from the origin that goes through the tangent point $x^E$; thus, in terms of Proposition 1, we only look at the frontier points. These firms operate, however, at different levels of technical efficiency, $\theta^C$ and $\theta^D$, respectively. Since the isocost representing the level of costs $w^C$ is closer to the origin than that of the expenditure level $w^D$, costs of firm C are smaller than those of firm D, while firm C is more profitable than firm D. Since $0b^C/0a^C > 0b^D/0a^D$, $\theta^C > \theta^D$, larger technical efficiency is associated with larger profits for points forming the frontier in profit-technical efficiency space. This proves that such frontier is upward sloping.

Proposition 2. The higher the allocative efficiency the higher the profit. For any arbitrarily chosen level of technical efficiency, the closer the input combination to the optimal one (i.e., the larger the allocative efficiency) the larger the profit will be.

Proof. Eq. (3) suggests that in terms of Fig. 2 (all three firms are equally technically efficient) costs solely depend on allocative efficiency. Moreover, the smaller the allocative efficiency the larger the expenditure. Keeping in mind that these firms produce the same output $y^A$, we conclude that for $\theta$-level of technical efficiency (again chosen arbitrarily) the larger the allocative efficiency the lower the costs and the larger the profit is; as allocative efficiency reaches its maximum (for firm C), the maximal profit is also achieved. Without loss of generality, this statement is true for any level of technical efficiency.

Fig. 3. Profit at different output levels.

Fig. 4. Relationship between technical efficiency and profit.
Remark 3. Proposition 2 is valid for different outputs.

Proof. Using the result of Remark 1, we claim that given technical and allocative efficiency the profit does not depend on output. It implies that regardless of produced output level for all equally technically efficient firms the same allocative efficiency purports the same level of profit. Recall pairs A and F, C and G from Fig. 3. Since allocative efficiency and profit of firms A and F are smaller than allocative efficiency and profit of firms C and G, the larger allocative efficiency is associated with larger profit.

\[ A \equiv F \]

\[ C \equiv G \]

\[ \text{Profit} \]

Fig. 5. Allocative efficiency in profit-technical efficiency space.


Proof. In Fig. 5 frontier is the locus of the maximum attainable profits as defined in Proposition 1. The firms A(F), B, and C(G) have the same technical efficiency level TE⁰; however, they have different profit levels: \( \pi_1 \), \( \pi_2 \), and \( \bar{\pi} \), respectively. The potential level of profit, which firms can reach is \( \bar{\pi} \). The closer the observation is to the frontier, the larger the profit is. As we recall from Fig. 2, the shift from firm A(F) to firm C(G) is only possible when the input-mix is changed; i.e., the allocative efficiency is improved. Thus, in Fig. 5, the shift from firm A(F) to firm B means an increase in allocative efficiency (distance \( AE^A \) is larger than distance \( AE^B \)), and further increase in allocative efficiency at the same level of technical efficiency is only possible up to the level of firm C(G), for which both profit and allocative efficiency are at the maximum. Without loss of generality, we can find as many firms between A and C as we may require, which ensures monotonic relationship between allocative efficiency and profit given technical efficiency. Since the level of technical efficiency is chosen arbitrarily, this Proposition holds on entire range of possible levels of technical efficiency. Thus, which is most remarkable, the distance from the observation to the frontier serves as a measure of allocative efficiency. Since we use pair A and C and pair F and G interchangeably, the proposition is valid for different levels of produced output.

Although two-inputs-one-output setup was used for illustration purposes because of its graphical simplicity, it is interesting to note that the generalization to the multivariate framework of the new approach is straightforward. In Fig. 2, the two-dimensional \( \langle x_1-x_2 \rangle \) space is substituted by the \( n \)-dimensional \( \langle x_1-\cdots-x_n \rangle \) space of \( n \) inputs, \( \mathcal{X} \). At the same time, the isonquant curve \( L(y^A) \) representing one output is substituted by an isocost hypersurface of dimension \( (n-1) \) representing \( m \) outputs, \( \mathcal{Y} \). The isocosts become hyperplanes. Although impossible to build a picture, the multivariate setup works just the same as the two-inputs-one-output case, which we used here for the sake of proof.

\[ \text{IsoqL}(\mathcal{Y}) = \{ \mathcal{X} : \exists \lambda \in L(\mathcal{Y}), \lambda \mathcal{X} \in L(\mathcal{Y}) \text{ for } \lambda \in [0,1] \} \]

is equal to the number of inputs minus one.
Multivariate framework is only used to retrieve technical efficiencies, whereas the profit-oriented distance, the new measure of allocative efficiency is found in the two-dimensional space (profit-technical efficiency) irrespective of inputs and outputs dimensionality.

To summarize, we have defined a new way of estimating allocative efficiency, specifically, this is the profit-oriented distance to the frontier in profit-technical efficiency space.

2.3. Assessments of restrictiveness of Assumptions A1–A3

A1. Might be a restrictive assumption in empirical applications (see for example Varian, 1992, pp. 15–16 for possible violations of this assumption). From a theoretical point of view this assumption is justified by the existence of competitive input and output markets.

A2. The assumption on equal input prices is necessary to ensure that isocosts are parallel to each other. Moreover, it guarantees that the difference between revenues and costs increases by the same proportion as revenues. The following reasoning justifies this assumption. We implicitly assume that input prices clear the resource market in terms of quality (e.g., Blanchard et al., 1992). This means that if input prices \( \bar{w} \) are larger than \( w \) then their quality is better and these resources are more productive than those available at price \( w \). That is why resources are input prices \( \bar{w} \) represent different technology, which is superior. On the other hand, if input prices \( w' \) are lower than \( w \), the quality of resources available at \( w' \) is lower and technology, which they represent, is inferior. In the Monte-Carlo experiment we relax the assumption of equal input prices in order to test whether the new approach still yields valid estimates of allocation efficiency. The assumption of equal output price is based on the same reasoning. That a firm is facing larger output prices implies that outputs are of better quality, which would purport a superior technology. And vice versa, that a firm is receiving lower output prices, implies worse quality of output, which is produced using inferior technology. See also the discussion in Kuosmanen et al. (2006).

A3. This assumption is not restrictive, but rather provides flexibility. Indeed, to avoid size effects, different relative measures of profit can be used in empirical applications: ratio of revenues to costs, ratio of difference between revenues and costs or to revenues etc.

3. Monte-Carlo simulation

To analyze the tractability of the new approach to measuring allocative efficiency, we conducted several Monte-Carlo experiments. The traditional approach requires data on inputs, outputs, and input prices; for the new approach data on inputs, outputs, and profits are needed. Generating simulated data, which contains all required information allows computing and comparing allocative efficiency score using both approaches.

3.1. Empirical implementation of the traditional approach

The traditional approach can be used when input prices are known. For \( m \) outputs, \( \mathcal{Y} \), and \( n \) inputs, \( \mathcal{X} \), under technology \( T \) such that

\[
T = \{ (\mathcal{X}, \mathcal{Y}) : \mathcal{X} \text{ can produce } \mathcal{Y} \},
\]

we measure input-oriented technical efficiency as the greatest proportion that the inputs can be reduced and still produce the same outputs:

\[
F^i(\mathcal{Y}, \mathcal{X}) = \inf \{ \lambda \in \mathbb{R}^+ : \lambda \mathcal{X} \text{ can still produce } \mathcal{Y} \}.
\]

In the Monte-Carlo simulations as well as in empirical part based on real data, we employ the Data Envelopment Analysis (DEA). For \( K \) observations, \( M \) outputs, and \( N \) inputs an estimate of the Farrell Input-Saving

\[\text{See Kneip et al. (1998, 2003) for statistical properties of DEA.}\]
Measure of Technical Efficiency can be calculated by solving a linear programming problem for each observation $j$ ($j=1,\ldots, K$):

$$
\hat{\text{TE}}_j = \hat{F}^j(y,x|C) = \min \left\{ \lambda : \sum_{k=1}^{K} z_k y_{km} \geq y_j, \sum_{k=1}^{K} z_k x_{kn} \leq x_j, z_k \geq 0 \right\},
$$

for $m=1,\ldots, M$ and $n=1,\ldots, N$. Note that superscript $i$ stands for input orientation while $C$ denotes constant returns to scale. Other returns to scale are modeled adjusting process operating levels, $z_k$s (see Färe and Primont, 1995 for details).

When input prices and quantities are given, we can calculate the total costs and the minimum attainable cost (solve linear programming problem) and then compute an estimate of cost efficiency for each observation $j$ ($j=1,\ldots, K$) as in Eq. (2):

$$
\hat{C}^j(y,x,w|C) = \min \left\{ \sum_{n=1}^{N} w_{jn} x_{jn} : \sum_{k=1}^{K} z_k y_{km} \geq y_j, \sum_{k=1}^{K} z_k x_{kn} \leq x_j, z_k \geq 0 \right\},
$$

for $m=1,\ldots, M$ and $n=1,\ldots, N$. Here, $z_k$s denote individual intensity variables, which are obtained when solving the efficiency problem. We refer to the residual of technical and cost efficiencies as Input Allocative Efficiency, which can be computed for each observation $j$ ($j=1,\ldots, K$) as:

$$
\hat{\text{AE}}_j = \frac{\hat{C}^j(y,x,w|C)}{\hat{F}^j(y,x|C)}.
$$

3.2. Empirical implementation of the new approach

As mentioned above, the main virtue of the new approach is that we do not necessarily need input prices for measuring allocative efficiency. Technically, we need profit-oriented distances to the frontier in the profit-technical efficiency space. We take advantage of the technical efficiency estimates (denoted by TE) obtained as in Eq. (6) and profitability measure (denoted by Pr) to calculate profit-oriented distance to the frontier in profit-technical efficiency space as theoretically derived in Section 2. Solving linear programming problem for each observation $j$ ($j=1,\ldots, K$), the allocative efficiency is given by:

$$
\hat{\text{AE}}_j = \max \left\{ \theta : \sum_{k=1}^{K} z_k^\theta \text{Pr}_k \geq \text{Pr}_j, \sum_{k=1}^{K} z_k^\theta \text{TE}_k \leq \text{TE}_j, z_k^\theta \geq 0 \right\}.
$$

Similar to Eqs. (6) and (7), $z_k$s denote individual intensity variables for a particular (for observation $j$) linear programming problem.

Although we present Monte-Carlo evidence based on two-inputs-one-output setup, we have explained in the theoretical part that the new approach is able to accommodate multivariate cases as well as the traditional one. The new approach resides on the calculation of technical efficiency scores, which are easily computed in the multivariate framework.

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7 It remains to be studied empirically, which returns that scale assumption has to be made here. This can be done by applying, for example, nonparametric test of returns to scale developed by Simar and Wilson (2002). To reduce computational burden, we leave this to be investigated further and are interested now in how the new measure works under CRS assumption.
3.3. Design of the Monte-Carlo experiments

In the experiments we simulate data on inputs, output and input prices. From these variables we compute revenues by setting the output price to a numeraire equal to one and obtain profits by dividing the difference of revenues and costs by revenue (see Assumption A3). We obtain two datasets by this simulation: dataset (1) with information on inputs, output, and input prices which is required for the traditional approach and dataset (2) with information on inputs, output, and profit revenue share which is required for measuring AE with the new approach. Note that although in dataset (2) profit is calculated using input prices, they are however not used in retrieving allocative efficiency measures with the new approach.

In each of the Monte-Carlo, we define a production process which uses two inputs to produce one output. Data for the $i$th observation in each Monte-Carlo experiment were generated using the following algorithm.

(i). We chose output elasticities of two inputs to be 0.2 and 0.8.\(^8\)
(ii). Draw $x_1 \sim (\Phi + \lambda \cdot \text{uniform})$; uniform on the interval $[0;1]$.
(iii). Draw $r \sim \text{uniform}$; uniform on the interval $[0;8]$. This is meant to be an experimental ratio of used inputs.
(iv). Set $x_2 = rx_1$.
(v). Choose $\varepsilon$. In doing so, we allow the ratio of inputs in each Monte-Carlo trial to vary on the interval $[\varepsilon; 8 - \varepsilon]$.
Therefore, we obtain enough variation of inefficient combinations of inputs, or in other words, enough variation of allocative inefficiency.
(vi). Draw $u \sim N(0, \sigma_u^2)$ and set ‘te_drawn’ equal to $\exp(-u)$.
(vii). Generate output data assuming translog production function, which will contain inefficiency component:\(^9\)
\[
\ln y_i = 0.2 \ln x_{1i} + 0.8 \ln x_2 + \frac{1}{2} \gamma_{11} \ln^2 x_{1i} + \frac{1}{2} \gamma_{22} \ln^2 x_{2i} + \gamma_{12} \ln x_{1i} \ln x_{2i} + \text{te\_drawn}_i,
\]
\[
\text{(10)}
\]
The chosen parameter values ensure homogeneity of degree one. We run the simulations with $N=100$ and with $N=400$.
(viii). Draw price of input $x_1$: $w_1 \sim (\varphi + \psi \cdot \text{uniform})$, uniform on the interval $[0;1]$. The price of input $x_2$ is calculated as $w_2 = \theta w_1$—we want to keep the ratio of input prices constant to have the isoquants parallel (recall Fig. 2).\(^10\)
(ix). Set profit as output (we set output price equal to 1) minus cost and this is divided by output. This is a relative measure as in Assumption A3: (revenue–costs)/costs.
(x). Run DEA to obtain traditional allocative efficiency as in Eq. (8).
(xi). Run DEA to obtain the new measures of allocative efficiency using technical efficiency drawn in step (vi) as in Eq. (9).
(xii). Solve for technical efficiency as in Eq. (6), and run DEA to obtain the new measure of allocative efficiency using these solved technical efficiency scores.
(xiii). Calculate rank correlation coefficient between allocative efficiency estimates based on traditional and our approaches.
(xiv). Repeat steps (i) through (xiii) $L$ times.

In each of our experiments we set $\Phi=1$, $\lambda=7$, $\varphi=1$, $\psi=0.05$, $\gamma_{11}=0.01$, $\gamma_{22}=0.01$, and $\gamma_{12}=0.002$.\(^11\) In order to look at different variabilities of inappropriately chosen ratios of inputs, we set $\varepsilon=0.5$, 1, 2. With $\varepsilon=2$, variability of allocative efficiency expected to have been reduced considerably—range becomes $[2;6]$; and vice versa, $\varepsilon=0.5$ ensures very large variability—range increases to $[0.5;7.5]$. We conduct three sets of experiments setting $\sigma_u^2$ to 0.0025,
From Tables 1–6 it is clearly seen that in all three cases the DEA estimates the drawn technical efficiency scores fairly accurately—the rank correlation coefficient (Corr4) is close to one. This is an expected outcome since we do not assume a stochastic term in the production output generation (step (vii) of the experiment). The same argument applies to the rank correlation coefficient between allocative efficiency calculated in step (xi) and that calculated in step (xii) (Corr3). Thus, there is not much difference in using the true or the estimated technical efficiency in the new approach. However, what is most interesting to us is the rank correlation between technical efficiency calculated in Eq. (6) and that drawn in step (vi).

\[ \epsilon = 0.5, N=100. \]

Standard errors are in *italics*.

\[ a \] Corr1 is the rank correlation between allocative efficiency calculated in step (x) and that calculated in step (xi),

\[ b \] Corr2 is the rank correlation between allocative efficiency calculated in step (x) and that calculated in step (xii),

\[ c \] Corr3 is the rank correlation between allocative efficiency calculated in step (xi) and that calculated in step (xii),

\[ d \] Corr4 is the rank correlation between technical efficiency calculated in Eq. (6) and that drawn.

\[ \epsilon = 1, N=100. \]

Notes from Table 1 apply.

0.025, and 0.25; this ensures covering a plausible range of standard deviations of technical efficiency.\(^{12}\) In each experiment we ran \( L = 500 \) Monte-Carlo trials.\(^{13}\)

### 3.4. Results

From Tables 1–6 it is clearly seen that in all three cases the DEA estimates the drawn technical efficiency scores fairly accurately—the rank correlation coefficient (Corr4) is close to one. This is an expected outcome since we do not assume a stochastic term in the production output generation (step (vii) of the experiment). The same argument applies to the rank correlation coefficient between allocative efficiency calculated in step (xi) and that calculated in step (xii) (Corr3). Thus, there is not much difference in using the true or the estimated technical efficiency in the new approach. However, what is most interesting to us is the rank correlation between allocative efficiency estimates from the traditional and our new approach (Corr1 and Corr2). Corr1 has been computed with the estimates of allocative efficiency based on “true” technical efficiency while Corr2 has been computed with the estimates of allocative efficiency based on estimated values of technical efficiency. As previously mentioned, the rank correlation between these measures is quite high (Corr3). We argue that it is more appropriate to draw conclusions from Corr2 since we do not know the “true” technical efficiency in practice.

\(^{12}\) Using a different experiment, Greene (2005) obtains estimates of technical efficiency with standard deviations from 0.09 to 0.43.

\(^{13}\) The simulation is programmed in SAS 9.1.3; computationally, one run with \( N=100, L=500 \) takes about seven hours a Pentium IV processor running at 3 GHz. Thus, we defined relatively few parameter constellations in the performed experiment.
The first observation worth mentioning is that when variability of sub-optimal ratios decreases ($\varepsilon$ increases), our method is less successful in yielding similar estimates as the traditional one. Hence, under current CRS assumption (see footnote 6) our method deteriorates in terms of exactness when “true” allocative efficiency is not heterogeneous.

Furthermore, the results show that our approach is robust with respect to variance of the drawn technical efficiency, $\sigma^2_u$. Looking closely at correspondent ratios, one can notice that for the same $\theta$s Corr2 is increasing when $\sigma^2_u$ increases, whereas for other $\theta$s Corr2 decreases when we increase $\sigma^2_u$; however, the changes are minor. The same argument applies to the standard deviation of Corr2. This implies that for different levels of $\sigma^2_u$ distributions of Corr2 are virtually the same. The skewness of the variable Corr2 is always negative and is about $-0.6$ which means that the distribution of Corr2 is skewed to the left and more values are clustered to the right of the mean. Kurtosis is about 0.6, but it varies more than the skewness; it increases with increase of $\sigma^2_u$. Kernel density estimates of Corr2 for the case $\theta=0.75$ are shown in Fig. 6. Note that we use the Gaussian kernel function and the Sheather and Jones (1991) rule to determine the “optimal” bandwidth.

The results are better when the sample size is increased to 400 (Tables 4–6). However, the improvement does not change our main conclusions based on the experiments with sample size 100. As expected, standard deviations of rank coefficients are almost halved when the sample size is quadrupled.

### 4. Empirical illustration of the new approach

In order to illustrate the usefulness of the new approach to measuring allocative efficiency we apply it to firm-level data containing information on inputs, output and profitability but not on input prices. The aim of this exercise is to perform a cost efficiency analysis for different size categories, which implies that cost, technical, and allocative

<table>
<thead>
<tr>
<th>$\sigma^2_u$</th>
<th>0.0025</th>
<th>0.025</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.75</td>
<td>1</td>
<td>1.25</td>
</tr>
<tr>
<td>Corr1</td>
<td>0.814</td>
<td>0.5782</td>
<td>0.3386</td>
</tr>
<tr>
<td>Corr2</td>
<td>0.8155</td>
<td>0.5837</td>
<td>0.3448</td>
</tr>
<tr>
<td>Corr3</td>
<td>0.9939</td>
<td>0.9948</td>
<td>0.9938</td>
</tr>
<tr>
<td>Corr4</td>
<td>0.9455</td>
<td>0.9449</td>
<td>0.9443</td>
</tr>
</tbody>
</table>

$\varepsilon=2$, $N=100$.

Notes from Table 1 apply.
efficiency have to be estimated. It is obvious that traditional approach to allocative efficiency measurement would preclude such analysis due to the fact that input prices are not available.

4.1. Data

The micro-data is drawn from the German Cost Structure Census\textsuperscript{14} of manufacturing for the year 2003. The Cost Structure Census is gathered and compiled by the German Federal Statistical Office (Statistisches Bundesamt). Enterprises are legally obliged to respond to the Cost Structure Census; hence, missing observations due to non-response are precluded. The survey comprises all large German manufacturing enterprises which have 500 or more employees. Enterprises with 20–499 employees are included as a random sample that is representative for this size category in a particular industry. For more information about cost structure census surveys in Germany, we refer the reader to Fritsch et al. (2004). Our sample comprises only enterprises from the chemical industry. The measure of output is gross production. This mainly consists of the turnover and the net-change of the stock of the final products.\textsuperscript{15}

The Cost Structure Census contains information for a number of input categories.\textsuperscript{16} These categories are payroll, employers’ contribution to the social security system, fringe benefits, expenditure for material inputs, self-provided equipment, and goods for resale, for energy, for external wage-work, external maintenance and repair, tax depreciation

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\textsuperscript{14} Aggregate figures are published annually in a panel-series 4, series 4.3 of the Cost Structure Census (various years). For more details on the Cost Structure Census, see Appendix A1.

\textsuperscript{15} We do not include turnover from activities that are classified as miscellaneous such as license fees, commissions, rents, leasing etc. because this kind of revenue cannot adequately be explained by the means of a production function.

\textsuperscript{16} Though the production theory framework requires real quantities, using expenditures as proxies for inputs in the production function is quite common in the literature (see for example Paul et al., 2004; Paul and Nehring, 2005).
of fixed assets, subsidies, rents and leases, insurance costs, sales tax, other taxes and public fees, interest on outside capital as well as “other” costs such as license fees, bank charges and postage, or expenses for marketing and transport.

Some of the cost categories which include expenditures for external wage-work and external maintenance and repair contain a relatively high share of reported zero values because many firms do not utilize these types of inputs. Such zeros make the firms incomparable and, thus, might bias the DEA results. In order to reduce the number of reported zero input quantities, we aggregated the inputs into the following categories: (i) material inputs (intermediate material consumption plus commodity inputs), (ii) labor compensation (salaries and wages plus employer’s social insurance contributions), (iii) energy consumption, (iv) user cost of capital (depreciation plus rents and leases), (v) external services (e.g., repair costs and external wage-work), and (vi) “other” inputs related to production (e.g., transportation services, consulting, or marketing).

Profit is computed as one minus the total costs divided by the turnover. Since the DEA requires positive values, we standardize the profit measure to the interval (0,1) by adding the minimum profit and dividing this by the range of profits.¹⁷

¹⁷ In this way profit is in relative terms as the new approach requires.
4.2. Empirical results

Fig. 7 shows profitability plotted against estimated technical efficiency. Remarkably, a frontmer, as could be theoretically expected from Proposition 1, indeed exists. It is also noteworthy that at a certain level of technical efficiency profitability ranges greatly suggesting quite some variation in allocative efficiency (as firms A(F), B, and C(G) in Proposition 3) and that the profits are bounded from above. Moreover, the frontier is positively sloped as was stated in the first theoretical part of this paper. Interestingly, Fig. 7 suggests that even being 100% technically efficient does not necessarily imply being 100% allocatively efficient.

Table 7 presents basic descriptive statistics for cost, technical, and allocative efficiency scores for the entire sample and by size categories. Note that averages are weighted.18

The average cost efficiency for the entire sample is quite low.19 The results from Table 7 imply that an “average” German manufacturing firm could have reduced its costs by 47.7% by operating on the efficiency frontier. It is clear that there are significant opportunities to reduce operating costs, however a deeper and dynamic analysis is required to judge about the long-run performance of the industry. When we compare the performance of firm in different size categories, small firms are better than average and moreover, they are outperforming larger counterparts. This better performance of “small” firms, however, is not very pronounced.

The most remarkable result comes from the decomposition of cost efficiency. The most severe efficiency loss happens as a result of technical inefficiency, which averages as high as 34.4%. This means that most German manufacturing firms are rather not successful in operating close to the production frontier. As in the case with cost efficiency, small firms are performing on average better than larger firms. The second source of loss in efficiency comes from allocative inefficiency. The average allocative inefficiency is 19.4%, implying that German manufacturing firms are not achieving cost minimizing combination of inputs. Since the figure is virtually the same for all-sized firms, the argument holds correspondingly.

Note however, that this analysis of allocative efficiency is not exhaustive so far, since it is only based on cross-sectional data. Anandalingam and Kulatilaka (1987, p. 143) state:

Farrell (1957) noticed that “there remains the question of whether a high price (i.e. allocative) efficiency is necessarily desirable.” He argued that in a dynamic context, firms may over invest in factors in the short-run in order to achieve long-run goals and thus appear to be allocatively inefficient in a static sense.

18 As shown by Färe and Zelenyuk (2003) the simple averages of technical and allocative efficiency scores are misleading and weighted averages have to be adopted instead.

19 The studies that analyze cost efficiency in financial sector are frequent and appear persistently in refereed journals in various years; see, for instance Akhavein et al. (1997), Lang and Welzel (1998) and up-to-date analysis by Weill (2004) and Cummins and Rubio-Misas (2006). The major reason for that is availability of firm-level input prices in the industry. While cost efficiency analysis is particularly important for manufacturing, we cannot compare our estimates to those of other studies, because such studies are extremely rare.
Thus, a dynamic analysis is required to assess the ability of German manufacturing firms to adopt an optimal combination of inputs.

5. Concluding remarks

Allocative inefficiency, introduced in the seminal work by Farrell (1957), has important implications from the perspective of the firm. On the other hand, the existing empirical evidence on the extent and determinants of allocative efficiency within and across industries is rather limited. The major reason is that the traditional approach to assessing allocative efficiency requires input prices. However, input prices are rarely accessible, which per se, precludes the analysis of the allocative efficiency.

In this paper, a new method is developed which enables calculating allocative efficiency without knowing input prices. This indicator is derived as the profit-oriented distance to the frontier in profit-technical efficiency space. Thus, besides input and output quantities, only the profits of the firms are needed for calculating allocative efficiency. The necessary assumptions for the proposed methodology could be regarded as restrictive, however, we believe that these assumptions do not considerably reduce the usefulness of the new approach for empirical studies, and that this method opens doors to many interesting applications. A simple Monte-Carlo experiment was performed to check the validity of the new methodology. Both the new and the traditional method lead to comparable measures of efficiency for individual firms. This is confirmed by the high-rank correlation coefficients between allocative efficiency estimates based on both approaches for different parameter constellations. Moreover, the new approach proved to be quite robust with respect to variance of true technical efficiency.

Finally, we applied the new approach to a sample of about 900 enterprises from the German chemical industry to highlight the usefulness of our method for obtaining allocative efficiency measures when input prices are not available. The conducted cost efficiency analysis would not be possible by means of the traditional approach. Empirical results suggest a fairly low level of cost efficiency. Technical inefficiency is mainly the cause of the latter observation. Moreover, the empirical analysis shows that smaller firms are performing better than larger ones. Furthermore, German chemical manufacturing firms are on average allocatively inefficient, i.e., do not choose an optimal combination of inputs. The latter finding, however, is based on mere cross-section and dynamic analysis has to be done in this direction.

Table 7
Descriptive statistics of cost, technical, and allocative efficiency

<table>
<thead>
<tr>
<th>Size category</th>
<th>Type of efficiency</th>
<th>N</th>
<th>Mean</th>
<th>S.D.</th>
<th>Min</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 49 employees</td>
<td>Cost</td>
<td>205</td>
<td>0.570</td>
<td>0.117</td>
<td>0.183</td>
<td>0.498</td>
<td>0.566</td>
<td>0.640</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Technical</td>
<td>205</td>
<td>0.703</td>
<td>0.150</td>
<td>0.357</td>
<td>0.600</td>
<td>0.683</td>
<td>0.809</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Allocative</td>
<td>205</td>
<td>0.816</td>
<td>0.082</td>
<td>0.513</td>
<td>0.758</td>
<td>0.807</td>
<td>0.867</td>
<td>1</td>
</tr>
<tr>
<td>50–99 employees</td>
<td>Cost</td>
<td>225</td>
<td>0.559</td>
<td>0.117</td>
<td>0.237</td>
<td>0.493</td>
<td>0.560</td>
<td>0.637</td>
<td>0.883</td>
</tr>
<tr>
<td></td>
<td>Technical</td>
<td>225</td>
<td>0.735</td>
<td>0.161</td>
<td>0.405</td>
<td>0.596</td>
<td>0.704</td>
<td>0.848</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Allocative</td>
<td>225</td>
<td>0.771</td>
<td>0.089</td>
<td>0.432</td>
<td>0.743</td>
<td>0.794</td>
<td>0.860</td>
<td>1</td>
</tr>
<tr>
<td>100–249 employees</td>
<td>Cost</td>
<td>212</td>
<td>0.531</td>
<td>0.098</td>
<td>0.252</td>
<td>0.488</td>
<td>0.559</td>
<td>0.619</td>
<td>0.922</td>
</tr>
<tr>
<td></td>
<td>Technical</td>
<td>212</td>
<td>0.666</td>
<td>0.144</td>
<td>0.415</td>
<td>0.596</td>
<td>0.681</td>
<td>0.785</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Allocative</td>
<td>212</td>
<td>0.810</td>
<td>0.084</td>
<td>0.528</td>
<td>0.770</td>
<td>0.819</td>
<td>0.868</td>
<td>1</td>
</tr>
<tr>
<td>250–499 employees</td>
<td>Cost</td>
<td>114</td>
<td>0.560</td>
<td>0.091</td>
<td>0.255</td>
<td>0.471</td>
<td>0.553</td>
<td>0.594</td>
<td>0.836</td>
</tr>
<tr>
<td></td>
<td>Technical</td>
<td>114</td>
<td>0.690</td>
<td>0.131</td>
<td>0.430</td>
<td>0.585</td>
<td>0.662</td>
<td>0.753</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Allocative</td>
<td>114</td>
<td>0.816</td>
<td>0.083</td>
<td>0.587</td>
<td>0.750</td>
<td>0.808</td>
<td>0.861</td>
<td>0.990</td>
</tr>
<tr>
<td>500–999 employees</td>
<td>Cost</td>
<td>77</td>
<td>0.486</td>
<td>0.096</td>
<td>0.325</td>
<td>0.482</td>
<td>0.558</td>
<td>0.624</td>
<td>0.835</td>
</tr>
<tr>
<td></td>
<td>Technical</td>
<td>77</td>
<td>0.604</td>
<td>0.160</td>
<td>0.325</td>
<td>0.589</td>
<td>0.686</td>
<td>0.815</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Allocative</td>
<td>77</td>
<td>0.816</td>
<td>0.086</td>
<td>0.620</td>
<td>0.726</td>
<td>0.803</td>
<td>0.864</td>
<td>1</td>
</tr>
<tr>
<td>more than 1000 employees</td>
<td>Cost</td>
<td>72</td>
<td>0.494</td>
<td>0.118</td>
<td>0.360</td>
<td>0.459</td>
<td>0.522</td>
<td>0.594</td>
<td>0.966</td>
</tr>
<tr>
<td></td>
<td>Technical</td>
<td>72</td>
<td>0.617</td>
<td>0.152</td>
<td>0.460</td>
<td>0.563</td>
<td>0.653</td>
<td>0.787</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Allocative</td>
<td>72</td>
<td>0.805</td>
<td>0.074</td>
<td>0.600</td>
<td>0.748</td>
<td>0.794</td>
<td>0.858</td>
<td>0.966</td>
</tr>
<tr>
<td>total</td>
<td>Cost</td>
<td>905</td>
<td>0.523</td>
<td>0.108</td>
<td>0.183</td>
<td>0.487</td>
<td>0.557</td>
<td>0.626</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Technical</td>
<td>905</td>
<td>0.656</td>
<td>0.151</td>
<td>0.325</td>
<td>0.591</td>
<td>0.682</td>
<td>0.803</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Allocative</td>
<td>905</td>
<td>0.806</td>
<td>0.084</td>
<td>0.432</td>
<td>0.750</td>
<td>0.806</td>
<td>0.866</td>
<td>1</td>
</tr>
</tbody>
</table>

* Averages are due to Färe and Zelenyuk (2003).
References


