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Measuring performance heterogeneity within groups – a two-dimensional approach

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We introduce a two-dimensional measure for the heterogeneity of performance within groups. It leads to a much more differentiated description of heterogeneity than alternative measures and it is relatively robust with regard to extreme values of small units ('outliers').

I. Introduction

Economic units are typically quite heterogeneous with respect to their economic performance. For example, while some firms operate at the technological frontier and earn high profits, others lag considerably behind and are scarcely able to survive competition. Conventional means of measuring such dispersion of performance, for example, using the range or the standard deviation, disregard the relative importance of the economic units. Furthermore, these traditional measures tend to be rather vulnerable with regard to extreme values. We propose a new measure of heterogeneity of economic units based on a two-dimensional approach and taking into account both the relative size of economic units and dispersion of performance. An illustrative example demonstrates the robustness of this measure regarding outliers.

II. The Performance Distribution Curve and Measurement of Aggregate Performance

As a fictive example, Fig. 1 shows a graphical exposition of a sample of economic units, such as a group of households or the firms of an industry, showing diverging levels of economic performance.[1] Performance may describe different issues, for instance, profit, productivity, or efficiency. In this graph, the units are arranged according to their performance in descending order, starting with the best performing unit. This unit constitutes the 100% benchmark for measuring the relative performance of the other entities in the respective group; that is, the performance of a unit is measured in relation to the performance of the best performing group member with the value of 100% in this distribution. The length of the line for each unit is equivalent

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[1] This exposition is inspired by diagrams in Salter (1969). Salter displayed productivity levels of firms in ascending order, starting with the least efficient firm.

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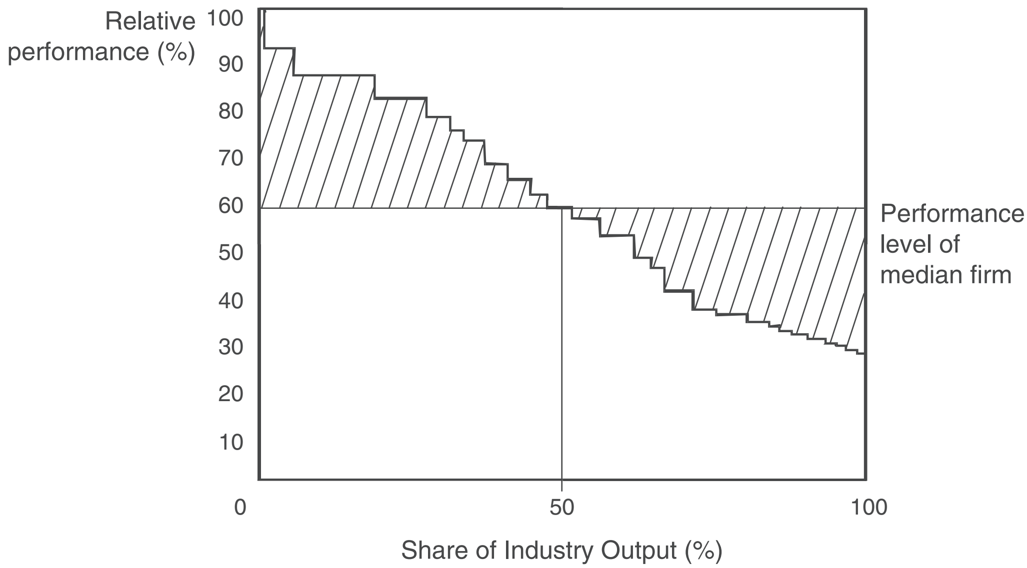

Fig. 1. The performance distribution curve

to its relative size that may be measured by its share of employment, assets, gross production, or turnover in the whole group (Fig. 1). Small entities are represented by short lines and larger entities by longer lines. The resulting curve provides an informative portrayal of the distribution of performance within the respective group. The value of about 30% for the worst performing unit in Fig. 1 indicates that its performance level is about 70% below that of the performance leader.[2] The total range of the performance distribution is calculated by subtracting the percent value of the least performing group member from 100%.

The distribution of performance levels of individual units points to a basic issue. When taking the unit with the highest value of the performance indicator as a benchmark for assessing the performance of the other group members, this performance level should be somewhat representative of the whole group. Best performing 'outliers,' lead to figures of relative performance that may be judged inadequate or wrong. Therefore, measures for the distribution of performance within groups should not give too much weight to extreme values. This implies that the range between the minimum and the maximum value is probably not well suited to this means of measurement. Compared to the range, other conventional measures of heterogeneity such as the standard deviation and the coefficient of variation have the advantage that they are using the information for all members of the respective group, not only the minimum and maximum. However, these measures do not account for the relative size of the individual units. It makes a huge difference whether the best performing unit has a large share or whether it plays only a marginal role.

III. A Two-Dimensional Approach for Measuring Heterogeneity within Industries

Our measure of heterogeneity is based on the performance distribution curve. It is the area between the performance distribution curve and the median value of the performance indicator in the sample. This heterogeneity area (ha) is defined as follows:

$$ha = \sum_{1}^{I} |p_i - p_m| s_i,$$

where p_i $(0 \leq p_i \leq 1)$, denotes the relative performance level of a unit i $(i = 1, \ldots, I)$ as a percentage and p_m is the relative performance level of the median unit. This median is defined according to the measure of relative size that is used for constructing the curve. The relative size of a unit as a percentage is denoted by s_i $(0 \leq s_i \leq 1)$. Our measure can have values

[2] In an empirical analysis for German manufacturing industries during the period from 1992 to 2001, we find that the median efficient production unit is about 59% of the maximum efficiency level and that the minimum efficiency level is, on average, about 38.5%. There are, however, large differences between the various industries with regard to these figures. See Fritsch and Stephan (2003) for details.

Table 1. Numerical examples of alternative measures of heterogeneity

	Absolute performance/relative performance/share					
	A	B	C	D	E	F
Unit I	100/100/10	100/100/30	100/100/5.55	100/100/2.78	100/50/9.9	100/100/9.9
Unit II	90/90/10	90/90/7.78	90/90/5.55	90/90/2.78	90/45/9.9	90/90/9.9
Unit III	80/80/10	80/80/7.78	80/80/5.55	80/80/2.78	80/40/9.9	80/80/9.9
Unit IV	75/75/10	75/75/7.78	75/75/5.55	75/75/2.78	75/37.5/9.9	75/75/9.9
Unit V	70/70/10	70/70/7.78	70/70/5.55	70/70/2.78	70/35/9.9	70/70/9.9
Unit VI	65/65/10	65/65/7.78	65/65/50	65/65/75	65/32.5/9.9	65/65/9.9
Unit VII	60/60/10	60/60/7.78	60/60/5.55	60/60/2.78	60/30/9.9	60/60/9.9
Unit IIX	50/50/10	50/50/7.78	50/50/5.55	50/50/2.78	50/25/9.9	50/50/9.9
Unit IX	40/40/10	40/40/7.78	40/40/5.55	40/40/2.78	40/20/9.9	40/40/9.9
Unit X	30/30/10	30/30/7.78	30/30/5.55	30/30/2.78	30/15/9.9	30/30/9.9
Unit XI	–	–	–	–	200/100/1	5/5/1
Mean	66.0	66.0	65.0	66.0	39.09	60.45
Median	67.5	75.0	65.0	65.0	32.5	67.5
Range	0.70	0.70	0.70	0.70	0.85	0.95
Standard deviation	0.2071	0.2071	0.2071	0.2071	0.2167	0.2641
Heterogeneity area (ha)	0.1700	0.1955	0.0935	0.0473	0.1528	0.1746
Ha 10–90	0.1373	0.1831	0.0330	0.0031	0.0510	0.0997
Ha 25–75	0.1025	0.0422	0.0014	0	0.0367	0.0363

between 0 and 0.5. It will be zero if all units have the same performance value and, conversely, it is be 0.5 if half of the group performs at 100% and the other half has a performance of 0%. The measure takes into account two dimensions: the relative performance level as well as the relative size of the units. This two-dimensional character makes it relatively robust with regard to extreme values. In contrast to other measures of heterogeneity such as the standard deviation or the coefficient of variation, our area measure is sensitive to relative size of the group members, for example, whether the high performing units have a relatively large share or constitute only a marginal share of the group. Since performance is expressed as the percent deviation from the highest attained performance level and size is measured as the percent share of the group, this indicator is independent from any absolute figures and the values can be directly compared between groups. The measure is also relatively robust with regard to small units with extreme values that may not be considered as being representative for the group.

IV. Numerical Examples

Some numerical examples (Table 1) may demonstrate the properties of our heterogeneity measure. Let us assume that a group consists of ten units (unit I – X) with relatively normally distributed performance levels as displayed in column A of Table 1. For the sake of simplicity, we assume that the performance value of the best performing unit is 100 so that absolute and relative performance is identical. Each of the units in column A constitutes 10% of the group. In columns B to D, these units have the same performance levels as in column A. However, their percent shares vary. Whilst the values of the mean and range are not affected by these differences, and remain constant in columns A to D, the median and the heterogeneity area react to the variation of shares within the group. In column B, the best performing unit has a share of 30%, whilst the rest is equally distributed among the remaining units, which attain about 7.78% each. Compared to the constellation of column A, diversity within the group has increased because a larger proportion is now performing well above the average. Accordingly, the value of the heterogeneity area is higher. However, as soon as the share of the performance leader is more than 50%, it represents the median performance level and the area measure of heterogeneity assumes relatively low values. If a relatively large unit is performing at about the average level, the heterogeneity area also tends to be relatively small (see columns C and D).

In order to demonstrate the properties of our heterogeneity measure with regard to extreme values, column E shows a constellation in which a further unit (unit XI) is included in the group that is characterized by performance twice as good as the best unit in the 'old' group. Since outliers tend to be quite small, we assume that the additional unit has a share of only 1%. The other ten units have an equal share of 9.9% each. The inclusion of an additional case with a higher performance level has two effects. First, whilst the absolute performance of the 'incumbents' remains constant, the values for their relative performance decrease due to the higher reference level given by the additional unit. They are, therefore, closer together than in the reference constellation shown in column A. However, in contrast to this greater homogeneity among the old units, the inclusion of a unit with better performance also leads to greater dispersion and, consequently, greater heterogeneity within the new group. In the case displayed in column E, our area measure gives greater weight to the first of the two effects, indicating an overall decrease of heterogeneity within the sample. The main reason for this result is the small share of the extreme case (unit XI) which is only 1%. If this share were larger, the inclusion of this case could result in a rising value of the indicator. The values for the range and for the standard deviation do not account for the relative size of the extreme value and point towards an increase in the heterogeneity level within the sample. Adding a small outlier, with relatively low performance (unit XI in column F) to the initial group of ten units (column A) leads to more heterogeneity, according to all three measures. The reason why our area measure indicates increasing heterogeneity here is that the additional case does not lead to a change of the reference point for calculating relative performance. However, the increase of the area measure is only 2.7% compared to an increase in the range of 11.8% and in the standard deviation of about 4.3%. This demonstrates the robustness of our measure with regard to small outliers.

To obtain a heterogeneity indicator that is even less affected by outliers, the measure may be calculated without including the best and the worst performing units. Table 1 shows the values without the upper and the lower 10% (ha 10–90) and 25% (ha 25–75). The difference (often relatively large) between these indicators and the values for the complete heterogeneity area indicates that in many cases the greater part of the heterogeneity is due to the tails of the distribution.

V. Conclusions

We have presented a two-dimensional approach to measuring performance heterogeneity within a sample of economic units that accounts for the relative size of these units. This new measure leads to a more precise description of performance heterogeneity and is, in particular, more robust with regard to outliers.[3]

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[3] For an analysis of technical efficiency within industries on the basis of this measure, see Fritsch and Stephan (2003).